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On the Determination of the Solar Parallax from the Parallactic Inequalities in the Longitude of the Moon, and on the Correction to Hansen's Coefficient of the Annual Equation.

By E. Neison, Esq.

It has long been well known that if it were possible to determine from observations the true value of the coefficient of the parallactic inequality in the longitude of the Moon, it would furnish a most advantageous method of ascertaining the distance of the Sun with great accuracy. For the coefficient of the parallactic inequality is equal to the solar parallax multiplied by nearly fifteen, so that an error of 0″·15 or a little less than one-sixth of a second of arc in the value of the parallactic inequality would produce an error of only 0″·01 in the value of the solar parallax. A number of attempts have been made to ascertain the real value of the parallactic inequality in the longitude of the Moon, by comparing the observed place of the Moon with the place assigned to it by the tables, and, from the difference between observation and theory, deducing by known methods a correction to the value of the parallactic inequality employed in the tables. In this way various values have been obtained for the coefficient of the parallactic inequality, ranging between 123″·5 deduced by Burckhardt, and 126″·46, the last result deduced by Hansen. These correspond to values of the solar parallax of 8″·63 to 8″·90.

The tables with which the observations were compared were up to 1862 unsatisfactory, which rendered the determination of the corrections to the tabular values far less certain than was desirable, unless a very long series of similar observations could be obtained. Since that period Hansen's Tables have been generally employed, and being much more complete and accurate than any previous tables, it much facilitates the comparison of observation and theory and the deduction of corrections to the values of the inequalities employed in the tables.

From the latest investigation it would appear that the value of the parallactic inequality employed by Hansen in his tables is 2″·0 too great, and ought to be reduced to 124″·46, corresponding to a solar parallax of 8″·67. Unfortunately, this apparent correction is not really the true correction to the coefficient of the parallactic inequality, but is really the sum of the true correction to the coefficient of the parallactic inequality *plus* a periodical variation in the apparent semi-diameter of the Moon, due to varying contrast between the brightness of the Moon's limb and the brightness of the sky it is seen against. When the parallactic inequality tends to decrease the longitude of the Moon, then this variation in the semi-diameter tends to increase it; whilst, when the parallactic inequality increases the longitude, then the variation in semi-diameter decreases it. Accordingly, the observed value of the parallactic inequality is always less than the truth.

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These drawbacks have now been known for some time, and they have been considered by most competent judges as a fatal obstacle to this method of determining the solar parallax. In this view I by no means concur, on the double ground that, not only are there theoretical objections to the sufficiency of the methods which have hitherto been used to deduce the value of this inequality from observation, but that there are more advantageous methods of accurately determining this inequality than by transits of the Moon's limb. One of these methods is about to be undertaken by Professor Winnecke, and, I am convinced, will lead to the determination of the coefficient of the parallactic inequality to within $0''\cdot 1$.

My present purpose, however, is not the consideration of the best methods of obviating these objections to the determination of the solar parallax from the parallactic inequality, but to suggest a method of obtaining the desired result without making use of this inequality, but by employing another which is not liable to the systematic errors peculiar to the parallactic inequality. This term is the term of long period, whose argument is the angular distance between the Sun and the perigee of the Moon's orbit. In my notation it is

$$\theta - \alpha,$$

where θ is the angular distance between the mean longitudes of the Sun and Moon, and α denotes the Moon's mean anomaly. It is what Delaunay denotes by

$$\mathrm{D} - l.$$

This term has a period of nearly 400 days, and, being a term of long period, is independent of any errors which may be contained in any of the innumerable terms of short period in Hansen's Tables. It is likewise entirely independent of the uncertainty as to the true value of the Moon's semi-diameter, or of any variations in this semi-diameter. Since Hansen's Tables were introduced, the argument of this term has undergone about thirteen revolutions, and the lunar perigee has nearly completed two revolutions, so that it ought to be possible to determine with some accuracy the value of the coefficient of this term from the fifteen years' observation of the Moon which are now available. It ought to be possible to determine the value of this term to within $0''\cdot 05$, and before long, when the theory of the terms of long period in the Moon's motion is more satisfactorily known, and when a few more years' observation are at my disposal, I think the value of this inequality could be determined from observation to within $0''\cdot 02$.

The exceptional freedom of these classes of terms of long period from systematic errors of observation, and their complete independence of the effect of the errors in the terms of short period forming the very great majority of the perturbations of the

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Moon, render it very easy to determine their value from observation with very small error indeed.

In this term of long period the solar parallax is multiplied by the factor 2·118; it is therefore less favourable than the 14·25 of the well-known parallactic inequality. If, however, the value of this term could be determined by observation to within $0''\cdot05$, as I surmise, it would give the value of the solar parallax to $0''\cdot023$, whereas at present it is uncertain to five times this quantity, and may be anything between $8''\cdot740$ and $8''\cdot915$.

With the view of determining the solar parallax in this manner, I have deduced the value of this term of long period from the fifteen years' observations with the Greenwich Transit Circle between 1862 and 1876. I should have liked to have had eighteen years' continuous observations, so as to include two complete revolutions of the perigee of the Moon's orbit, and when I can obtain the observations for the years 1877–1879 I shall repeat the work, including these years, so as to obtain a perfectly symmetrical investigation.

During this period rather over 1,500 observations of the Moon were made with the Greenwich Transit Circle, but a number of these had to be rejected as having been made under faulty conditions, as indicated by the notes appended to them. There remained 1,464 good observations which form the basis of the present investigation.

Let

$Q \sin q$ = the term of long period with the argument $(\theta-\alpha)$,

$W \sin w$ = the term of long period with the argument $(2\theta-2\alpha)$,

$M \sin \mu$ = the annual equation, μ being the mean anomaly of the Sun;

and put δQ, δW, δM for the corrections required by Hansen's tabular coefficients of these terms, these coefficients being Q, W, M.

Further, put

δl = the correction required by Hansen's value of the Moon's mean longitude,

$\delta\epsilon$ = the correction to Hansen's value of the Moon's mean longitude at the mean epoch of the observations.

Then Δ, the error in the Moon's tabular longitude, was equated to the above corrections in the form

$$\delta Q \sin q + \delta W \sin w + \delta M \sin \mu + \delta l + \delta\epsilon = \Delta.$$

The observations of each lunation were grouped together to facilitate the work, forming a system of 186 sets of equations of the above form.

The great difficulty in comparing Hansen's Tables with the Greenwich observations consists in the remarkable irregularity in the error in the Moon's mean longitude. Whilst the tables

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are rapidly falling away from the observed place of the Moon, the rate at which this occurs fluctuates in an inexplicable manner. This is well shown in Professor Newcomb's paper announcing the discovery of the new term due to the action of *Jupiter*. The two most marked breaks occur between the years 1867–1868 and 1870–1871. For this reason I have judged it best to determine $\delta\epsilon$ separately for each year from the observations of that year.

The terms depending on the errors in the coefficients of the annual equation, and on the perigee term (w = twice the distance of the Sun from the perigee of the lunar orbit), have been introduced for theoretical reasons. It is never safe to assume that the errors of a term of half the period of the particular term which may be being determined will destroy each other in the course of a short number of observations. For this reason the term depending on δW has been introduced. A similar remark applies to the annual equation which has a period only one-ninth shorter than the parallactic term to be determined. My main reason for introducing this term was, however, the fact that my researches had shown me that Hansen's coefficient was faulty, so that it might be expected to exercise some influence.

The final equations were reduced to the system

$$\begin{aligned} 926{\cdot}15\,\delta Q + 21{\cdot}30\,\delta W + 122{\cdot}74\,\delta M &= +\;\;29{\cdot}16'', \\ 33{\cdot}54\,\delta Q + 923{\cdot}37\,\delta W + 66{\cdot}61\,\delta M &= -1016{\cdot}29, \\ 90{\cdot}71\,\delta Q + 59{\cdot}01\,\delta W + 948{\cdot}84\,\delta M &= +\;\;25{\cdot}82, \end{aligned}$$

which on solution gives the result

$$\begin{aligned} \delta Q &= +0{\cdot}175'', \\ \delta W &= +0{\cdot}101, \\ \delta M &= -1{\cdot}061. \end{aligned}$$

At first sight it would appear that Hansen's value of the annual equation ought to be increased by over a second of arc, but it must be remembered that the effect of determining $\delta\epsilon$ in the foregoing manner merges the effect of the year's variation in the Moon's mean longitude into the correction to the annual equation. The value of δM really includes both the total correction to the annual equation and a portion of the correction to Hansen's motion in mean longitude.

In forming sin q it has been found convenient to put

$$\sin(\theta-\alpha) = -\sin(-\theta+\alpha) = -\sin(l'-A),$$

where l' denotes the mean longitude of the Sun, and A the mean longitude of the lunar perigee. Q is therefore equal to the coefficient of the term

$$\sin(\theta-\alpha)$$

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with its sign reversed. The above result indicates therefore that Hansen's value of this coefficient ought to be increased by 0''·175, a very considerable increase. From Hansen's *Darlegung* it appears that he has assigned to Q, the coefficient of this term, the value (with its sign reversed)

$$\begin{aligned} Q &= +18''{\cdot}492 \times 1{\cdot}03573 \\ &= +19{\cdot}152 \end{aligned}$$

The effect of the correction deduced from observation would be to raise this to the value

$$Q + \delta Q = +19''{\cdot}327.$$

It remains to see what effect this change will have in altering Hansen's value for the solar parallax.

For convenience let it be supposed that the solar parallax is 8''·75 and that the Moon's mass is $\frac{1}{81{\cdot}5}$. Then, according to Delaunay, the coefficient of this term ought to be 18''·56, or, as I should make it from his data,

$$Q = 18''{\cdot}578,$$

for he appears to have underestimated the value of the remainder of his series.

Pontécoulant finds for this term the value 18''·41, or rather, from the analytical results of Pontécoulant's *Théorie de la lune*, this value can be deduced, employing the above data, which differs from that made use of by him. But Pontécoulant's coefficient is admittedly deficient in terms involving high powers of e and γ. Supplying these corrections, and allowing for an error which he afterwards detected, his coefficient becomes

$$Q = 18''{\cdot}54.$$

I have made sufficient advance in my own theory to be enabled to supply the principal part of this coefficient from my own researches. The coefficient has the value

$$\begin{aligned} Q = \Big\{ & \underset{+12''{\cdot}0786}{\frac{165}{32}\mathrm{m}} + \underset{+4''{\cdot}4367}{\frac{5997}{256}\mathrm{m}^2} + \underset{+1''{\cdot}4727}{\frac{98049}{1024}\mathrm{m}^3} + \underset{+0''{\cdot}6397}{\frac{8457917}{16384}\mathrm{m}^4} + \underset{+0''{\cdot}2953}{\frac{193186623}{65536}\mathrm{m}^5} \\ & + \underset{+0''{\cdot}1354}{\frac{87648232497}{524288}\mathrm{m}^6} + \ldots \Big\} b^2 e \cdot \frac{1-\lambda}{1+\lambda} = 19''{\cdot}0584\,\frac{1-\lambda}{1+\lambda}, \end{aligned}$$

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where m is as usual equal to $\frac{m}{1-m}$, λ is the mass of the Moon, and b^2 stands for the quantity of the second order $\frac{a}{a'}$, the ratio of the distances of the Sun and Moon. I am not quite sure of the last term being perfectly complete, as it is an order beyond the usual extent of my computations, and I have not yet quite completed this portion. It will be seen that this series converges appreciably quicker than that made use of by Delaunay. Adding to this coefficient the portion necessary to complete the preceding series (= 0''·0870), supplying from Delaunay the terms depending on the higher powers of e which I have not yet determined, and replacing λ by its value, the value of this coefficient becomes

$$Q = 18''{\cdot}5290.$$

All these values agree closely, and there is little doubt but that the value

$$Q = 18'{\cdot}550$$

is very close to the truth.

There is no difficulty in determining the coefficient of this term with any needed accuracy, for it would not be difficult to push the calculation to the order m^{10}, or, by numerical solution, to determine its value to 0''·001.

It appears, however, that the value indicated by observation exceeds this theoretical quantity by

$$+0''{\cdot}777,$$

and if this were really the true value of this coefficient as shown by the observations, it would correspond to a solar parallax of

$$9''{\cdot}117;$$

this is impossible.

It does not appear possible that there should be anything erroneous in the method of deducing this correction. It might be thought that the method of correcting the equations of condition for errors of longitude and epoch might introduce some systematic error. For this reason the observations were reduced anew in their unaltered form, determining both δl and $\delta \varepsilon$ from the total 1,464 observations. The values obtained were

$$\begin{aligned} \delta Q &= +0''{\cdot}187, \\ \delta W &= +0{\cdot}140, \\ \delta M &= -0{\cdot}586, \\ \delta l &= +0{\cdot}985, \\ \delta \epsilon &= 1864{\cdot}1. \end{aligned}$$

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These are practically identical with those previously obtained.

I think I shall be able to throw some additional light on this perplexing discrepancy when I have completed my determination of the theory of the terms of long period in the Moon's mean longitude. I am also reducing the Altazimuth observations of the Moon made at Greenwich, and the Washington Transit Circle observations.

I bring the question before the Society because it is important that such things should be made known, and because it throws a good deal of light on the high value of the solar parallax deduced by Hansen. When I commenced my inquiry I believed that I was the first to suggest the employment of this inequality of long period for the determination of the solar parallax. I think I am the first who has explicitly directed attention to its capabilities, and especially to its apparent freedom from systematic error, but I now believe that Hansen had made use of it for this purpose, and that it was mainly from this inequality that he determined the high value assigned by him to the solar parallax, and necessarily to the parallactic inequality. Hansen has nowhere explicitly referred to his having made use of this term for this purpose, but a paragraph in an early paper of his leaves little doubt on the matter, and this view is confirmed by several remarks in his *Darlegung*. The paragraph referred to occurs in a letter in the *Monthly Notices*, vol. xv., p. 9, and is as follows:—

'I may remark further that I have taken into consideration, not only the coefficient of the parallactic inequality itself, but also the largest of the remaining terms depending on the Sun's parallax, and that the aggregate of these terms may amount to the fifth part of the coefficient of the parallactic inequality.'

The first clause would seem clear enough, but the last renders it rather perplexing; for the sum of the three principal minor terms of the parallactic inequalities amount to not one-fifth, but two-fifths of the coefficient of the parallactic inequality. I am not certain, therefore, whether Hansen in this paragraph really alludes to this term of long period, or merely to the two other terms, which, having arguments of short period, are more naturally associated with the parallactic inequality, and the sum of the coefficients of these two terms would amount to one-fifth of the coefficient of this term. I have little doubt but that, subsequently, he did make use of this term of long period, from the tenor of his remarks in the *Darlegung*, though there is no explicit statement of the fact.

As Hansen's Tables now indicate a marked diminution of his coefficient of the parallactic inequality, his making use of this term would serve to explain how he came to render his value too great. Its apparent independence from systematic error would naturally lead to greater weight being attached to its results.

From the second system of equations it appears that Han-

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sen's value of the coefficient of the annual equation ought to be increased by more than half a second, although it is already a far larger value than has been arrived at by any other theorist. Hansen's value is 669″·880, whilst Delaunay finds 668″·910, and Pontécoulant 668″·932. I have not yet completed my own result, but I anticipate it will considerably exceed those of both Delaunay and Pontécoulant. It is, however, very desirable to determine the observed value of the annual equation, as it has an important bearing on the value of the parallactic inequality and variation. Moreover, its determination from observation is exposed to a systematic error tending to increase its apparent magnitude, and it is this augmented value which must be considered in discussing the observed place of the Moon with the view of deducing corrections to the other inequalities.

For this reason I was led to more carefully examine the correction required by Hansen's value of this coefficient. The observations were grouped into the three periods 1862–1867, 1868–1870, and 1871–1876, so as to avoid the effect of the sudden changes in the apparent mean motion which made its appearance between 1867–1868 and 1870–1871. These groups were then separately solved in the usual manner, and three accordant values were obtained for the correction to the coefficient of the annual equation. The final result from the three is

$$\delta M = -0''.7294 \pm 0''.0712,$$

and is the result of the reduction of 1,464 observations made with the Greenwich Transit Circle between the years 1862–1876. When thus augmented the coefficient of the annual equation becomes

$$M = -670'.609.$$

This result is greater than any which has been obtained from pure theory, unless Damoiseau's value of 673″·70 be considered as a purely theoretical value. Burckhardt, from observation, found the value 673″·3, but his Tables show this value to be too great. The Astronomer Royal, in his discussion of the Greenwich Observations between 1750 and 1850, was led to the value 670″·04, but considered this value probably too large. In the American Tables the coefficient 670″·3 is employed, and the comparison between the Washington Observations and these Tables shows that it cannot be seriously in error; whereas the comparison between Hansen's Tables and the Greenwich Transit Circle observations at once indicate that there is a discrepancy between the tabular and observed value of this coefficient.

The introduction of these corrections goes far to reduce the apparent irregularities in the motion of the Moon's mean longitude. The rate at which the tables are deviating from the true place of the Moon seems to be decreasing, and to correspond to a

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considerable extent to the effect of the faulty determination of the terms of long period in the Moon's mean motion arising from the action of the planet *Venus*.

In a subsequent communication I trust to be able to give a new determination of the parallactic inequality, in which the effects of the variation of the Moon's semi-diameter will be independently determined, and which shall be free from the objection which I think can be urged against the sufficiency of the theoretical basis of the previous determinations. I trust also to be able to remove the remaining irregularities in the apparent motion of Hansen's mean longitude, by tracing them to their source.

The Radiant Points of April 9–12. By W. F. Denning, Esq.

In recently drawing out a Table showing the number of fireballs and bright meteors observed on each day of the year, I was surprised at the large number recorded on April 11–12, July 27–30, November 19, and December 21; and, with the object of finding the chief showers in action during the first of these special epochs, I have just completed the projection of more than 700 shooting stars registered by Zezioli in the years 1867–70, and by other Italian observers in 1869 and 1872. The radiant points derived from these tracks numbered 21, of which several appear to be of more than ordinary richness. The positions of these centres, compared with previous determinations and with Mr. Greg's Catalogue of 1876, are given in the following Table. The major radiants are at π *Herculis*, δ *Ursæ*, and α *Draconis* with more than 40 meteors each. The first of these agrees with two showers found by Schiaparelli from Zezioli's observations, and with one seen by Mr. Corder and the writer in 1877. The average of the four independently assigned centres is at $245^{\circ}\cdot7 + 51^{\circ}\cdot5$, which accords very closely with the new position at $249^{\circ} + 51^{\circ}$. The maximum probably occurs on April 11, and the shower is no doubt connected (or included) with Mr. Greg's Draconids I at $263^{\circ} + 50^{\circ}$ (No. 47). The shower at δ *Ursæ* is equally well defined, and it had been already recognised by Mr. Greg as an active and persistent display with a centre at $180^{\circ} + 60^{\circ}$. The position at α *Draconis* (which also supplies several showers during the winter months) is not given in his Catalogue, but it falls near a strong radiant at $204^{\circ} + 56^{\circ}$ (No. 55). No. IV in the Table represents a conspicuous (though diffuse) centre of short meteors in *Coma Berenices*. The shower of Herculids (No. VII) appears to be sharply defined and distinct from several other April radiants near it, and this position may be regarded as very exactly determined from the following observations :—